# Lecture 21

Thursday, March 25, 2021 4:42 PM

* Prayer
* Spiritual thought
* Answering questions....

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## Line integral

Line int. of a scalar function

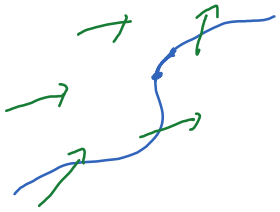
$$\int_C f(x,y)\,ds = \int_a^b f(x(t),y(t))\sqrt{x'(t)^2+y'(t)^2}\,dt$$

$$\int_C f(x,y,z)\,ds = \text{-----}$$

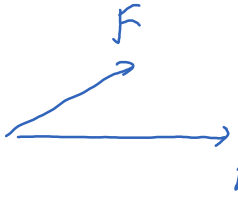
$$\int_C f(x,y)\,dx = \int_a^b f(x(t),y(t))\,x'(t)\,dt$$

$$\int_C f(x,y)\,dy = \text{-----}$$

The work done by $F$ along the curve $C$ is

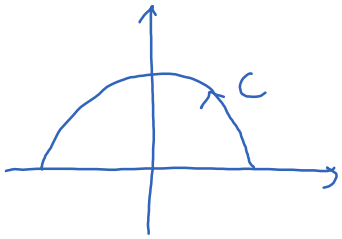


$$\int_C F\cdot\frac{r'(t)}{|r'(t)|}\,ds = \int_a^b F\cdot\frac{r'(t)}{|r'(t)|}\,|r'(t)|\,dt$$




$$= \int_a^b F\cdot r'\,dt = \int_C F\cdot dr$$

Ex:




$C: x = \cos t,\ y = \sin t,\ 0 \le t \le \pi$

$F(x,y) = \langle x, -y \rangle$

$$\int_C F \cdot dr = \int_C \langle x, -y \rangle \cdot r'(t)\, dt = \cdots\cdots$$

Fundamental thm for line int.

$$\int_C \nabla f \cdot dr = f(r(b)) - f(r(a))$$

$F = \langle 3 + 2xy,\ x^2 - 3y^2 \rangle$

$F = \langle 3 + 2xy,\ x^2 - 3y^2 \rangle$